Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 1 (Individual)

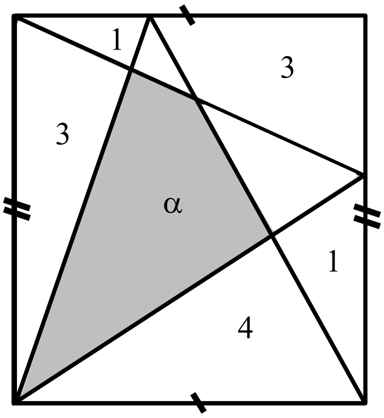
香港數學競賽 (2013 / 2014)

決賽項目 1 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求下圖中陰影部分的面積 α。

Determine the area of the shaded region, α , in the figure below.



圖一

Figure 1

α =

2. 如果 10 個不同的正整數的平均值是 2α，求這 10 個數中，最大的一個數 β 的最大可能值。

If the average of 10 distinct positive integers is 2α, what is the largest possible value of the largest integer, β , of the ten integers?

β =

3. 考慮兩組由正整數組成的有限數列：1, 3, 5, 7,⋯, β 和 1, 6, 11, 16, ⋯ , β + 1。求它們之間相同數字的數目 γ。

Given that 1, 3, 5, 7, … , β and 1, 6, 11, 16, … , β + 1 are two finite sequences of positive integers. Determine γ, the numbers of positive integers common to both sequences.

γ =

4. 若 $\log_2 a + \log_2 b \geq \gamma$，求 $a+b$ 的最小值 δ。

If $\log_2 a + \log_2 b \geq \gamma$, determine the smallest positive value δ for $a+b$.

$\delta =$

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 2 (Individual)

香港數學競賽 (2013 / 2014)

決賽項目 2 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求方程 $\sqrt{\left(x+\sqrt{x}\right)}-\sqrt{\left(x-\sqrt{x}\right)}=\sqrt{x}$ 的正實根 α。

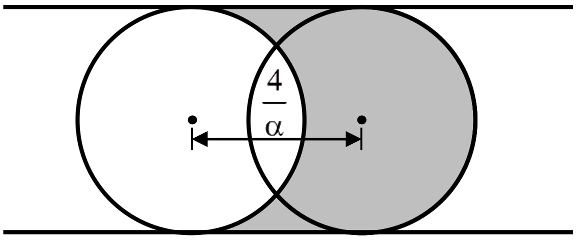
Determine the positive real root, α, of $\sqrt{\left(x+\sqrt{x}\right)}-\sqrt{\left(x-\sqrt{x}\right)}=\sqrt{x}$.

$\alpha =$

2. 下圖為兩個半徑為 4 的圓，其圓心相隔 $\frac{4}{\alpha}$。求陰影部分的面積 β。

In the figure below, two circles of radius 4 with their centres placed apart by $\frac{4}{\alpha}$. Determine the area, β, of the shaded region.

$\beta =$



3. 求正整數 γ 的最小值，以使得方程 $\sqrt{x}-\sqrt{\beta\gamma}=4\sqrt{2}$ 對 x 有正整數解。

Determine the smallest positive integer γ such that the equation $\sqrt{x}-\sqrt{\beta\gamma}=4\sqrt{2}$ has an integer solution in x.

$\gamma =$

4. 求 $\left(\left(\gamma^{\gamma}\right)^{\gamma}\right)^{\gamma}$ 的個位數 δ。

Determine the unit digit, δ , of $\left(\left(\gamma^{\gamma}\right)^{\gamma}\right)^{\gamma}$.

$\delta =$

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 3 (Individual)

香港數學競賽 (2013 / 2014)

決賽項目 3 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若數列 $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, \cdots, 10^{\frac{\alpha}{11}}$ 中所有數字的乘積為 1 000 000，求正整數 α 的值。

If the product of numbers in the sequence $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, \ldots, 10^{\frac{\alpha}{11}}$ is 1 000 000 , determine the value of the positive integer α .

$\alpha =$

2. 若 $\frac{\beta}{1\times2\times3}+\frac{\beta}{2\times3\times4}+\cdots+\frac{\beta}{8\times9\times10}=\alpha$ ，求 β 的值。

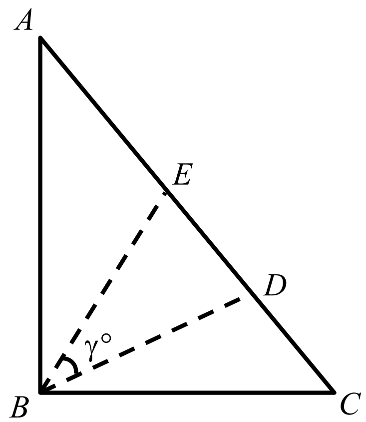
Determine the value of β if $\frac{\beta}{1\times2\times3}+\frac{\beta}{2\times3\times4}+\cdots+\frac{\beta}{8\times9\times10}=\alpha$.

$\beta =$

3. 在下圖的三角形 ABC 中，$\angle ABC = 2\beta°$，$AB = AD$ 及 $CB = CE$。設 $\gamma° = \angle DBE$ ，求 γ 的值。

In the figure below, triangle ABC has $\angle ABC = 2\beta°$, $AB = AD$ and $CB = CE$. If $\angle DBE = \gamma°$, determine the value of γ .

$\gamma =$



4. 考慮數列 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, ⋯，求首 γ 項的和 δ。

For the sequence 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, ⋯ , determine the sum δ of the first γ terms.

$\delta =$

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 4 (Individual)

香港數學競賽 (2013 / 2014)

決賽項目 4 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 $\frac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}=3\sqrt{\alpha}-6$，求 α 的值。

If $\frac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}=3\sqrt{\alpha}-6$, determine the value of α.

$\alpha =$

2. 考慮形如 $\frac{n}{n+1}$ 的分數，當中 n 是一個正整數。若同時把該分數的分子和分母減去 1，得出的分數是小於 $\frac{\alpha}{7}$，且大於 0，求這樣的分數的數目 β。

Consider fractions of the form $\frac{n}{n+1}$, where n is a positive integer. If 1 is subtracted from both the numerator and the denominator, and the resultant fraction remains positive and is strictly less than $\frac{\alpha}{7}$, determine, β, the number of these fractions.

$\beta =$

3. 一個等邊三角形和一個正六邊形的周長相同。若該等邊三角形的面積為 β 平方單位，求正六邊形的面積 γ (平方單位)。

The perimeters of an equilateral triangle and a regular hexagon are equal. If the area of the triangle is β square units, determine the area, γ, of the hexagon in square units.

$\gamma =$

4. 求 $\delta=\frac{3}{2}+\frac{5}{4}+\frac{9}{8}+\frac{17}{16}+\frac{33}{32}+\frac{65}{64}-\gamma$ 的值。

Determine the value of $\delta=\frac{3}{2}+\frac{5}{4}+\frac{9}{8}+\frac{17}{16}+\frac{33}{32}+\frac{65}{64}-\gamma$.

$\delta =$